FIRDesign

Finite Impulse Response
Digital Filter Design Utility

The Theory Behind

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Abbreviations

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References

[1] Oppenheim, A. R. and Schafer, R. W., Digital Signal Processing, Prentice Hall, 1975
[2] Rabiner, L. R. and Gold, B., Theory and Application of Digital Signal Processing, Prentice Hall, 1975
[3] Oppenheim, A. R. and Schafer, R. W., Discrete Time Signal Processing, Prentice Hall, 1989, 1999
[4] Rorabaugh, C. B., DSP Primer, McGraw-Hill 1998
[5] Schlichthärle, D., Digital Filters: Basics and Design, Springer 2000
[6] Lyons, R. G., Understanding Digital Signal Processing, Prentice Hall 1997
[7] Harris, F., On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform, Proc. IEEE, Vol. 66, No. 1, January 1978, pp. 51 – 83
[8] Rabiner, L. R., McClellan, J. H. and Parks, T., FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximation, Selected Papers in Digital Signal Processing, II, IEEE Press 1976, reprinted from Proc. IEEE, Vol. 63, April 1975, pp. 595 – 610
[9] McClellan, J. H., Parks, T. W. and Rabiner, L. R., A Computer Program for Designing Optimum FIR Linear Phase Digital Filters, Selected Papers in Digital Signal Processing, II, IEEE Press 1976, reprinted from IEEE Trans. Audio Electroacoust., Vol. AU21, Dec. 1973, pp. 506 – 526
[10] McClellan, J. H., Parks, T. W. and Rabiner, L. R., FIR Linear Phase Filter Design Program, Programs for Digital Signal Processing, IEEE Press 1979
[11] Rabiner, L. R. and McGonegal, C. A., FIR Windowed Filter Design Program – WINDOW, Programs for Digital Signal Processing, IEEE Press 1979
[12] Williams, C. S., Designing Digital Filters, Prentice-Hall 1986
[13] Hamming, R. W., Digital Filters, 3rd edition, Dover 1998
[14] Hamming, R. W., Numerical Methods for Scientists and Engineers, 2nd edition, Dover 1989
[15] Gold, B. and Rader, C. M., Digital Processing of Signals, McGraw-Hill 1969
[16] Lynch, P., The Dolph-Chebyshev Window: A Simple Optimal Filter, Monthly Weather Review, Vol. 125, April 1997, pp. 655 – 660

Modifications

1.0 Feb. 12, 2007 first issue

1. Introduction

1.1 Purpose

FIRDesign is a tool for the design of Finite Impulse Response filters. It offers three design methods, namely the sync plus window design, the frequency sampling design and the Remez exchange (Parks – McClellan) design. The program may be used to create source code in C or C++ for the filter coefficient tables to be imported into your DSP application software.

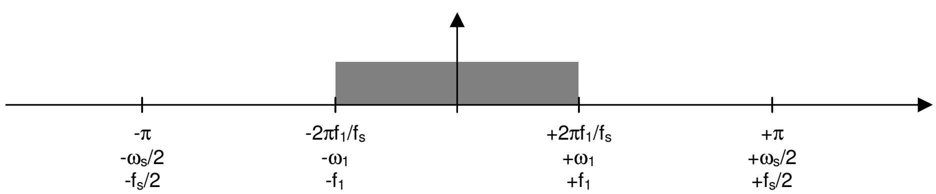
The purpose of this document is to outline the theory behind the various design methods and filter types.

This document is not complete. It may be completed at a later time.

2. Theory

2.1 Notation

2.1.1 Frequency Domain Scaling



In some cases, notably in connection with the Remez Exchange design method, we use the relative angular frequency

$\omega = f_1/f_s$.

2.1.2 Number of Taps

The number of taps of any filter is N. The number of intervals (unit delays) is N-1. For a filter with an odd number of taps, the indexes go from -(N-1)/2 via 0 to +(N-1)/2.

2.2 Number Format Ranges

The ranges represented by the various number formats are:

Name	Bytes		Range
Q7	1	signed char	0x80 (-1) to 0x7f (1-1/128)
Q15	2	signed short int	0x8000 (-1) to 0x7fff (1-1/32768)
Q31	4	signed long int	0x80000000L (-1) to 0x7fffffffL (1-1/2'147'483'648)

float	4	none	3.4E +/- 38 (7 digits)
double	8	none	1.7E +/- 308 (15 digits)
long double	10	none	1.2E +/- 4932 (19 digits)

2.3 Windows

2.3.1 Rectangular

The rectangular window definition is

w(k) = 1

2.3.2 Hanning

2.3.3 Hamming

2.3.4 Blackman

2.3.5 Dolph-Chebyshev

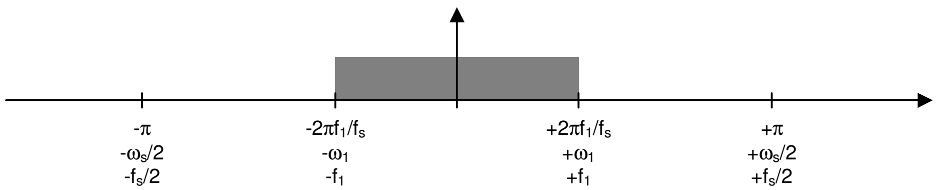
2.3.6 Kaiser

2.4 Impulse Responses

For the Window design method, we do need analytical expressions for the time domain impulse response of various filter types.

2.4.1 Lowpass Filter

The desired frequency response is:



The sampled values of the impulse response in the time domain are:

$$h_k = \frac{1}{2\pi}\int_{-\omega_1}^{\omega_1} e^{j\omega k}\, d\omega$$

$$h_k = \frac{1}{2\pi jk}\left[e^{j\omega_1 k} - e^{-j\omega_1 k}\right] = \frac{1}{\pi k}\left[\frac{e^{j\omega_1 k} - e^{-j\omega_1 k}}{2j}\right] = \frac{1}{\pi k}\sin(\omega_1 k)$$

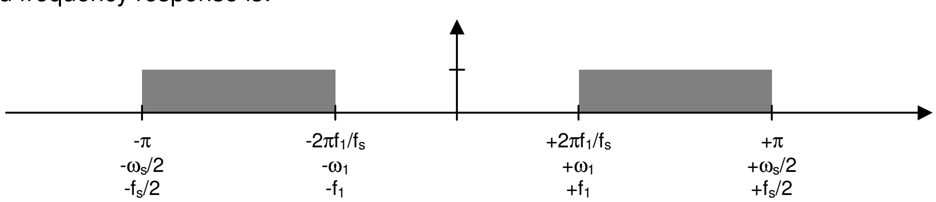
$$h_k = \frac{1}{\pi k}\sin(2\pi k\frac{f_1}{f_s}) \qquad k \neq 0$$

k = 0 is a special case which requires the use of the Bernoulli – L'Hôpital rule:

$$h_0 = \frac{2\pi\frac{f_1}{f_s}}{\pi}\cos(2\pi k\frac{f_1}{f_s}) = 2\frac{f_1}{f_s} \qquad k = 0$$

2.4.2 Highpass Filter

The desired frequency response is:



The sampled values of the impulse response in the time domain are:

$$h_k = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_1} e^{j\omega k} d\omega + \int_{+\omega_1}^{\pi} e^{j\omega k} d\omega\right]$$

$$h_k = \frac{1}{2\pi jk}\left[e^{-j\omega_1 k} - e^{-j\pi k} + e^{+j\pi k} - e^{+j\omega_1 k}\right] = \frac{1}{\pi k}\left[-\frac{e^{j\omega_1 k} - e^{-j\omega_1 k}}{2j} + \frac{e^{j\pi k} - e^{-j\pi k}}{2j}\right]$$

$$h_k = \frac{1}{\pi k}\left[-\sin(\omega_1 k) + \sin(\pi k)\right]$$

$$h_k = -\frac{1}{\pi k}\sin(2\pi k\frac{f_1}{f_s}) \quad k \neq 0$$

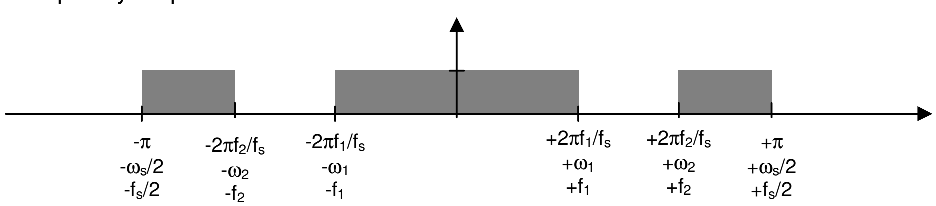
k = 0 is a special case which requires the use of the Bernoulli – L'Hôpital rule:

$$h_0 = -\frac{2\pi\frac{f_1}{f_s}}{\pi}\cos(2\pi k\frac{f_1}{f_s}) = -2\frac{f_1}{f_s} \quad k = 0$$

So the highpass filter coefficients are just the negative values of the lowpass filter coefficients.

2.4.3 Bandpass Filter

The desired frequency response is:



The sampled values of the impulse response in the time domain are:

$$h_k = \frac{1}{2\pi}\left[\int_{-\omega_2}^{-\omega_1} e^{j\omega k} d\omega + \int_{+\omega_1}^{+\omega_2} e^{j\omega k} d\omega\right]$$

$$h_k = \frac{1}{2\pi jk}\left[e^{-j\omega_1 k} - e^{-j\omega_2 k} + e^{+j\omega_2 k} - e^{+j\omega_1 k}\right] = \frac{1}{\pi k}\left[-\frac{e^{j\omega_1 k} - e^{-j\omega_1 k}}{2j} + \frac{e^{j\omega_2 k} - e^{-j\omega_2 k}}{2j}\right]$$

$$h_k = \frac{1}{\pi k}\left[\sin(\omega_2 k) - \sin(\omega_1 k)\right]$$

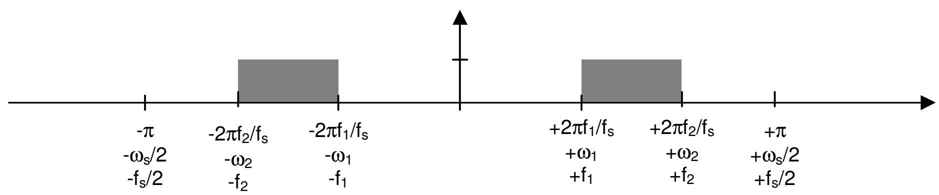
$$h_k = \frac{1}{\pi k}\left[\sin(2\pi k \frac{f_2}{f_s}) - \sin(2\pi k \frac{f_1}{f_s})\right] \quad k \neq 0$$

k = 0 is a special case which requires the use of the Bernoulli – L'Hôpital rule:

$$h_0 = \frac{1}{\pi}\left[2\pi \frac{f_2}{f_s}\cos(2\pi k \frac{f_2}{f_s}) - 2\pi \frac{f_1}{f_s}\cos(2\pi k \frac{f_1}{f_s})\right] = -2\frac{f_2 - f_1}{f_s} \quad k = 0$$

2.4.4 Bandstop Filter

The desired frequency response is:



The sampled values of the impulse response in the time domain are:

$$h_k = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_2} e^{j\omega k}\, d\omega + \int_{-\omega_1}^{+\omega_1} e^{j\omega k}\, d\omega + \int_{+\omega_2}^{+\pi} e^{j\omega k}\, d\omega\right]$$

$$h_k = \frac{1}{2\pi jk}\left[e^{-j\omega_2 k} - e^{-j\pi} + e^{+j\omega_1 k} - e^{-j\omega_1 k} + e^{+j\pi} - e^{+j\omega_2 k}\right]$$

$$h_k = \frac{1}{\pi k}\left[-\frac{e^{j\omega_2 k} - e^{-j\omega_1 k}}{2j} + \frac{e^{j\omega_1 k} - e^{-j\omega_1 k}}{2j} + \frac{e^{j\pi k} - e^{-j\pi k}}{2j}\right]$$

$$h_k = \frac{1}{\pi k}\left[\sin(\omega_1 k) - \sin(\omega_2 k) + \sin(\pi k)\right]$$

$$h_k = \frac{1}{\pi k}\left[\sin(2\pi k \frac{f_1}{f_s}) - \sin(2\pi k \frac{f_2}{f_s}) + \sin(\pi k)\right] \quad k \neq 0$$

k = 0 is a special case which requires the use of the Bernoulli – L'Hôpital rule:

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$$h_0 = \frac{1}{\pi}\left[2\pi\frac{f_1}{f_s}\cos(2\pi k\frac{f_1}{f_s}) - 2\pi\frac{f_2}{f_s}\cos(2\pi k\frac{f_2}{f_s}) + \pi\right] = \frac{2}{f_s}\left[f_1 - f_2 + \frac{f_s}{2}\right] \quad k = 0$$

2.4.5 Gaussian Differentiator

A good differentiator with no ringing can be obtained by using the first derivative of the Gaussian density as an impulse response:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\frac{d}{dx}p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \bullet -\frac{1}{2} \bullet 2\left(\frac{x-\mu}{\sigma}\right) \bullet \frac{1}{\sigma}$$

$$\frac{d}{dx}p(x) = \frac{-(x-\mu)}{\sigma^3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Or, for a simple case:

$$\frac{d}{dx}p(x) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \qquad \mu = 0 \qquad \sigma = 1$$

The impulse response must be causal and discrete, so

$$h(nT) = \frac{-1}{\sqrt{2\pi}}\left(\frac{2n - N + 1}{2}T\right) \bullet e^{-\frac{\left(\frac{2n-N+1}{2}T\right)^2}{2}} \qquad 0 \le n < N$$

For reasons of scaling, it does not make sense to keep the constant factor $\frac{1}{\sqrt{2\pi}}$, and T is set equal to 1, so the effective impulse response becomes:

$$h(n) = -1 \bullet \left(\frac{2n - N + 1}{2}\right) \bullet e^{-\frac{\left(\frac{2n-N+1}{2}\right)^2}{2}} \qquad 0 \le n < N$$

If $\sigma \neq 1.0$, still ignoring the constant factor:

$$h(n) = -1 \bullet \left(\frac{2n - N + 1}{2}\right) \bullet e^{-\frac{\left(\frac{2n-N+1}{2}\right)^2}{2\sigma^2}} \qquad 0 \le n < N$$

2.4.6 Moving Average

The moving average is a crude form of a low pass filter. All taps have equal weight. The impulse response is defined directly in the time domain:

$$h(nT) = \frac{1}{N} \qquad 0 \leq n < N$$

$$h(nT) = 0 \qquad N \leq n$$

3. Impulse Response Scaling

4. Sin(x)/x Compensation

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